

Difference between upper & lower probes can't be explained

1

If I know that proportion p of Red in Class I was chosen by a random process (over $0,1$), then I know a definite probability ($\frac{1}{2}$) that $p \geq \frac{1}{2}$; I will expect a bet on Red to pay off with the same frequency as a bet on Red when the known $p = \frac{1}{2}$.

Since my initial betting odds on Red will be the same in both cases, although I would pay for more info in the ~~second~~ first case (buy a sample) to determine "actual" proportion.

i.e. I have definite ~~Definite~~ Rank II probes in first case.

Similarly, if I know that lottery ticket was drawn from a set of tickets with equal chance of betting on Red or on Black, I know that a ticket drawn by such a method will pay off at the same frequency in the long run as a ticket to a drawing on Red with $p = \frac{1}{2}$.

Thus, I will not pay to switch ticket I for IV after the coin is flipped.

I value "the tickets which results from the flip" more than I value the ambiguous number of the pair before the flip. If there is going to be a flip between (I, II) and (II, IV), I would just as soon commit myself before the flip to (I or II) as to (III or IV), because after the flip I will be indifferent between I, and II, III, IV; I ~~or~~ II (whichever results from flip) will have risen in my estimation. If class I and II are both ambiguous,

R Y B G

1
B G
0 0 1 0

A Y B G

1 0 0 0
0 0 1 0
0 1 0 0
0 0 0 0

One ball: yellow or green.

One ball drawn from 50:50 Red or Black urn and placed in urn with other.

Cochrane

Good

Raiffa example

Smith

1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1

P*

P*

P
P
P
P
P
P
P
P

P
P
P
P

0 0 $\frac{1}{2}$ $\frac{1}{2}$

just as in a game, the pure strategy resulting from the operation of a random process is regarded as preferable to the same or a formerly-equivalent strategy not so chosen; the ticket chosen....

Think of (C) as an envelope containing a ticket chosen at random from bets on Red and bets on Black. Or: tickets are in numbered envelopes, or are face down and numbered; drawn by random process.

I am offered (a) a bet on Red, or (b) an envelope drawn by lot, either on Red or on Black.

I know the distribution of payoffs for bets like (C); I don't for (a). To be indifferent is to suggest that I assume that I know the parameter of the stochastic process by which the offer (a) was produced, or the parameters of the process by which the contents of the urn were determined.

Thus, RANDOMIZATION: produces bets (estimates) whose distribution of payoffs is known; i.e. such that I would give heavy odds on a prediction that the average payoff (e.g. error of estimate) would, in the long run, be confined within a narrow interval.

I would not, for non-randomized bets on ambiguous events. ~~For events with~~
~~prob. between .3 and .7~~ For such an event — if prob was "totally ambiguous" within ".3-.7" — I will act as if I were "almost sure" that minimum average payoff would be .3X (i.e. I would give high odds that it would not be ~~below~~ far below .3X) and I will give high odds that it will not be far above .7. Thus, my initial betting behavior w.r.t. these "prob limits" is similar to my bets on various average outcomes when I have a definite prob.

I will give high odds that average outcome r is between, say $p_* - \epsilon$ and $p_* + \epsilon$, where $0 \leq \epsilon < \max[p_*, (1-p_*)]$, $\epsilon \geq 0$.

Let A be the set of r s.t. $p_* - \epsilon \leq r \leq p_* + \epsilon$.

$$\bar{A} = 1 - A$$

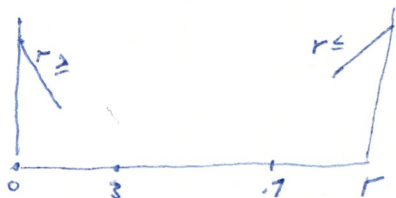
	$r \in A$	$r \in \bar{A}$
I	1	0
II	X	X

As ϵ increases (number of trials given), X $\rightarrow$ that ~~I~~ II increases $\rightarrow$ 1.

Or, for given ϵ , X increases $\rightarrow$ 1 as number of trials increases.

What is X as $\epsilon \rightarrow 0$?

Depends on N ; suppose N very large.



Three types of vagueness: (1) Interval between .7 and .3; (2) "Fuzziness" of .7 and .3 (differs with ambiguous events; may be negligible; e.g. if N is extremely large, may be "virtually certain" that $r \geq .3$; or may not, even then; i.e. may not give high odds that $r > p_* - \epsilon$ when ϵ is very small or 0. Problems of "estimating" p_* ; e.g. can "name" p_* so that I am willing to accept even bet that $r > p_*$ for large N . Then there are small ϵ such that I will demand only slightly favorable odds ~~to~~ but that $r > p_* + \epsilon$.

(3) Sampling error; r for small N may deviate from value for large N (by more than value for large N will differ from value for $2N$).

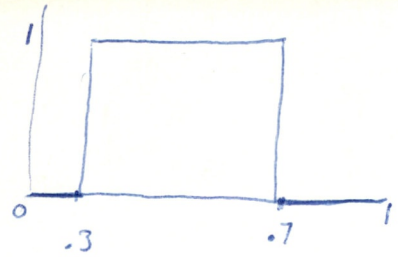
Last two correspond to estimation problems when $p_* = p^*$. Abstract from these: assume N very large, and opinions very definite (PI obeyed, limits "known precisely").

E.g. I "know" urn has either 3 or 7 Red balls; all Rand II probs over (.3, .7) are equally "reasonable," but ambiguous.

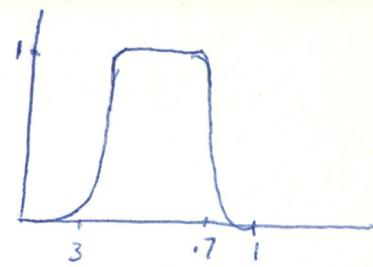


Savage: Good replaces one vague number with 2 vague numbers. No: different sorts of vagueness.

Bayesian:



or



But Ellsbergian doesn't act as if he had rectangular Rank II dist. over .3-.7.

e.g. he will not give 3:1 that $r < .6$ (or, $.3 \leq r \leq .6$). If he would, we could assign definite Rank II probs.

He may be indifferent between I-IV;

r between	.3-.4	.4-.5	.5-.6	.6-.7
I	1	0	0	0
II	0	1	0	0
III	0	0	1	0
IV	0	0	0	1
V	1	1	0	0
VI	0	1	1	0

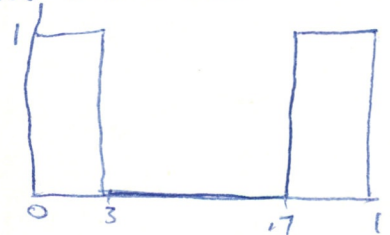
but

but not prefer V to I; or at least, not prefer VI to I. This is case in which he ~~is~~ finds prob. of r between .3 and .7 "completely ambiguous." He is "completely ignorant" in this range; he obeys Arrow Theorem, looks only at minimum and maximum. HURWICZ case.

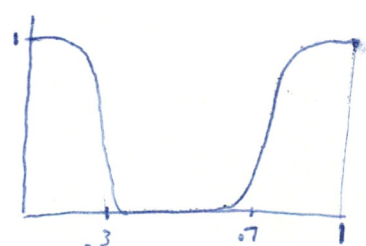
Rank II

Or, Hodges & Lehmann; he may have "best guess, dist." over .3-.7, but still $P < 1$. Thus, he may prefer VI > V > I, but not as much as if he assigned definite $\frac{1}{4}$ prob. to each interval above. i.e., he would prefer $\begin{matrix} 25 \\ R \end{matrix} \begin{matrix} 25 \\ Y \end{matrix} \begin{matrix} 25 \\ B \end{matrix} \begin{matrix} 25 \\ G \end{matrix}$ to VI, or $\begin{matrix} 25 \\ R \end{matrix} \begin{matrix} 25 \\ Y \end{matrix} \begin{matrix} 25 \\ B \end{matrix} \begin{matrix} 25 \\ G \end{matrix}$ to I, even though $I = II = III = IV$.

Invent above dist:



or



SHACKLE'S
POTENTIAL
SURPRISE!



Corresponding to

assume I obey Hurwicz rule within regions: P.S. = 0.
This is SHACKLE DECISION RULE!

RANK II Prob Dist, not
RANK I.

6

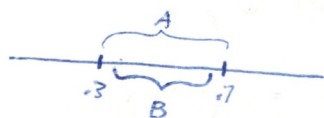
Shackle's mistake is to argue that $p.s. = 0$ for very large intervals for most events; even, in most extreme form, $p.s. = 0$ for $0 \leq r \leq 1$ for all events for which relevant frequency data is lacking.

But for some events, it is true that $p.s. = 0$ for an interval: such that I take into account only the best and worst values in that interval (with monotonic payoff for over parameter, this will be highest and lowest).
i.e. Solov Arrow conditions.

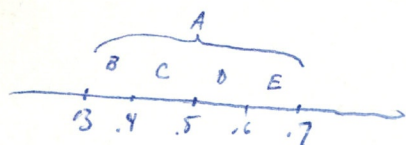
If ~~the best guess distribution~~ γ^0 may consist of a finite set of distributions, definite, $p > 0$,
e.g.; $[(.3, .7); (.7, .3)]$. If there is a "best guess" set of Rank II probabilities ~~between~~
over γ^0 — e.g. $(.5, .5)$ — then the set of betting weights may vary
continuously from $(.3, .7)$ to $(.7, .3)$ as p varies.

Thus, we reach my original results if (sufficient condition): person obeys Arrow Theorem, does use Shackle/Hemmings rule in betting on r , average outcome is a large series of independent trials under same conditions, within some interval $p_* \leq r \leq p^*$. Or (weaker suff. condition) if he uses $x/s/L$ rule within this interval (then he doesn't take only best + worst values into account.)

Implication: in estimating a parameter (e.g., r), suppose that I will give



high odds that $r \in A$, but will not give odds that $r \in B$ for any B , $A \supset B$, even when $A - B$ (i.e. $A \cap \bar{B}$) is fairly small.



	B	C	D	E	$\bar{A}$
I	1	0	0	0	0
II	0	1	0	0	0
III	0	0	1	0	0
IV	0	0	0	1	0
V	1	1	1	1	0
VI	.95	.95	.95	.95	.95
VII	.1	.1	.1	.1	.1
VIII	1	1	1	0	0
IX	0	1	1	1	0
X	0	0	0	1	0
XI	1	1	0	1	0
XII	.3	.3	.3	.3	.3

Assume $I = II = III = IV = VII$

$V = VI$

$VIII = IX = X = XI = XII$

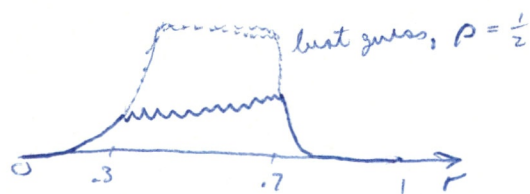
Thus, to give an interval estimate does not imply this sort of betting behavior in general; this implies "high confidence" in a particular interval, combined with very low confidence (unwillingness to bet except at very favorable odds) in any sub-interval, even a large one. E.g. "best guess" dist may be uniform within this interval; yet I demand favorable odds to bet that parameter lies within a sub-interval of half the measure of the interval (if I have the option of not betting) (if I must bet, I would be equally willing to bet for or against that half-interval).

(Savage axioms: suppose there is some event for which $\begin{pmatrix} \$12 & -10 \\ 8 & 0 \end{pmatrix}$. Then for any event such that $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, I should be willing to bet $\begin{pmatrix} \$12 & -10 \\ 8 & 12 \end{pmatrix}$

$\$12 + \-10 rather than not bet).

Thus, my Rank II "dist" may look like:

(A)



Definite (small) prob that $.3 > r$ ~~and~~ $.7 < r$ (say, 5-10%); all give odds that isn't.
say 10:1 or 20:1.

But ambiguity within;

I'll give, say, only slightly favorable odds that $.3 \leq r \leq .6$ (instead of 7:3) $\rightarrow$ say 3:2

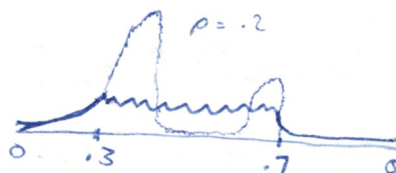
or that $.4 \leq r \leq .7$, even though, if I must bet, I am indifferent between

the two sub-intervals. I may even demand favorable odds for these bets, if I have the alternative of not betting; and I will do so to bet on, say, $.4 \leq r \leq .6$, or

$.3 \leq r \leq .5$, or $.5 \leq r \leq .7$. [Find an interval such that: a) I ~~would~~ am equally willing to bet on either half of the interval; b) I will demand favorable odds to bet on either half.]

Best guess needn't be uniform or symmetric:

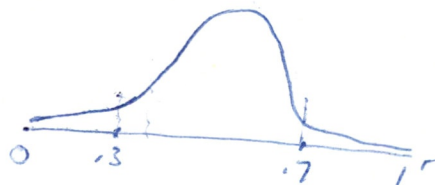
(B)



The lower P , the less the shape of the best-guess will matter.

All this is distinct from:

(C)



prob. ~~that~~ $.3 < r < .7$ } = .05

(D)

or

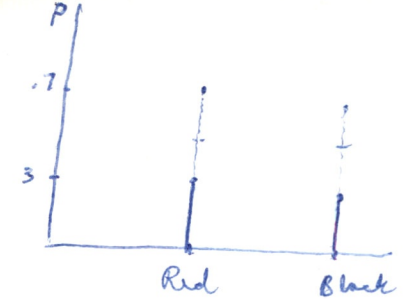


definite Rank II Distributions

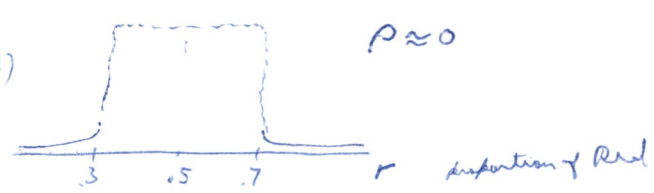
.75
.75
475
668
7125

Part I Prob. Dists.

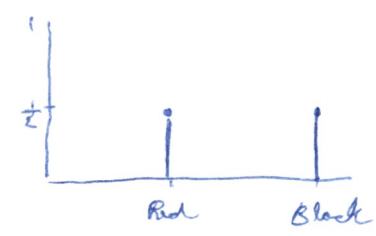
(A)



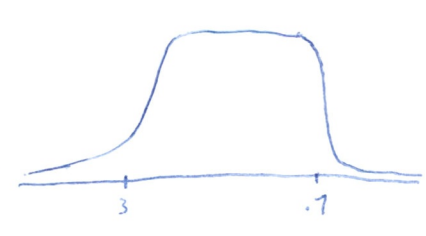
(4)



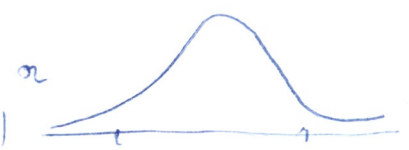
(D)



(D)



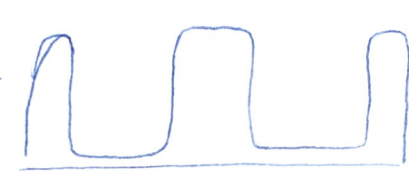
(E)



(E)



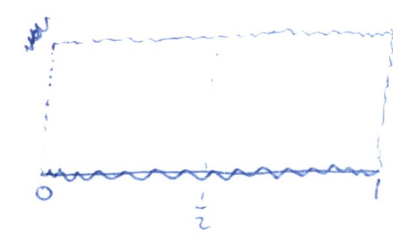
(F)



Value of info different in all these cases. R+S associate "vagueness" with cases like (D) or (E), as opposed to (C) or (F). But there are all definite Rank II distributions, leading to definite initial prior probs.

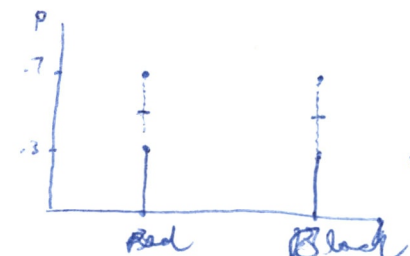
Ambiguous prior probs imply ambiguous higher rank probs, though not always like (A).

For suppose:



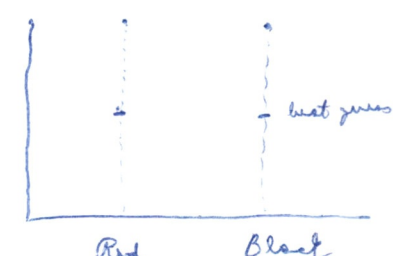
best guess

$$P = \frac{3}{5} ; \text{ or } P \approx 0, \alpha = .3$$



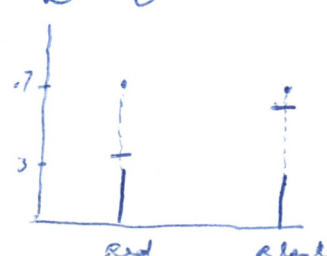
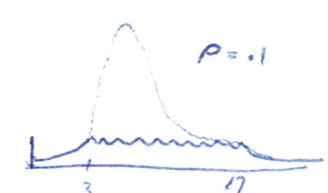
in terms of betting prospects

or



w.r.t. beliefs

OR:



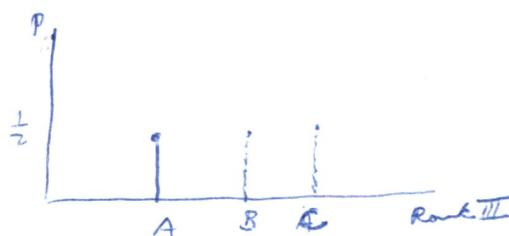
$P = .1$

Even Rank II dist. need not have just one ambiguous interval: 10

Rank II



$$\text{prob}(.5 < r < .7) = .5.$$



Thus, vagueness is not associated with Value of Info > 0 (as R+S suggest); it is manifested in initial betting quotients, before experiment; and, possibly in "ambiguity of" (lets on) the Value of Info. (if that depends on expectations of events, given actions).

If forecaster must express uncertainty without knowing what critical uncertainties are (e.g. is $p_i > k$; is p_i between 2 and 6?) he should give prob dist, or, equivalently, his full set of acceptable bets; not just a confidence interval: i.e. "I would give 19:1 that μ is between .7 and .9." Decision-maker may really want to know: "Would you give as much as 3:1 that $\mu > .75$?" Or; What odds would you demand ~~that~~ to bet on $\mu < .5$?

But it may be that there is an interval such that I would give high odds that parameter is within that interval, but I would not give nearly so high odds — perhaps, I would not give odds at all, but would demand favorable odds — to bet on any smaller interval. E.g. I may be equally willing to bet on either half of the interval, yet demand favorable odds (in utils) (i.e. demand better money odds than if I were betting on Heads) to bet on either half, conditionally.

A_1	A_2	$\bar{A}$
1	0	0
0	1	0
1	1	-19
0	0	0
2	-1	0
-1	2	0

Dostoevsky: Gambler: if red comes up 10 times, don't bet on red... or black.

Rumson: equit sides in your eye.

Difference between upper and lower ^{intuitive} probs can't be explained in terms of definite higher Rank probs; these will compound to give definite Rank I probs.

And difference doesn't necessarily result in upper + lower betting quotients (in utils; i.e. violation of Savage axioms).

Depends on how you bet, when upper + lower intuitive probs aren't equal.

Suppose P_* is the g.l.b. of the probs of unions of elements of a particular σ -scale, such that you can say definitely that $p(E) > P_*$. Let P^* be g.l.b. to probs such that you say definitely that $p(E) < P^*$.

BETS: Then, if N is an event such that $p(N) < P_*$, you will prefer to bet on E than on N . If $p(N) > P^*$, you will prefer to bet on N .

But what if $P_* < p(N) < P^*$?

1) There may be a definite $p(N')$, $P_* \leq p(N) \leq P^*$ such that when $p(N) < p(N')$, you prefer to bet on E and when $p(N) > p(N')$ you prefer to bet on N . Then your betting quotients B_* and B^* are equal; your betting probability $b(E)$ is appraisable, $= p(N)$, ~~even though~~ your "intuitive probability" is not.

2) You may reach this result by a convention; e.g. arbitrarily pick $p(N') = \frac{P_* + P^*}{2}$ (or $= \frac{P_* + P^*}{4}$, or $\frac{P_* \cdot P^*}{4}$, etc.)

In either case, you obey the Savage axioms.

3) a) Flip a ^{fair} coin between betting on E or N

b) Pick a $p(E)$ by random selection between P_* and P^* ; if $p(E) > p(N)$ bet on E ; etc.

These violate P1 (unless you do this once for $p(E)$ and then remember and use the result in all subsequent bets). However, the expected value of $p(E)$ will be $\frac{P_* + P^*}{2}$; you will obey stochastic axioms, and $p(E)$ can be inferred statistically from your choices.

4) As above, a) or b), but with non-symmetric randomization.

E.g. weighted more heavily toward lower prob for the event on which you were betting.

Violate both P1 and P2; a bias against ambiguity.

5.2) Minimax between P_* and P^* . (Hurwicz) (Bross) (Good). Then $b_* = P_*$, $b^* = P^*$
 b) or Restricted Bayes solution (H+L)
 Violate P2

12

Correspondence between intuitive prob. and betting prob:

Post: $P_{1*} > P_{2*} \Rightarrow b_{1*} > b_{2*}$

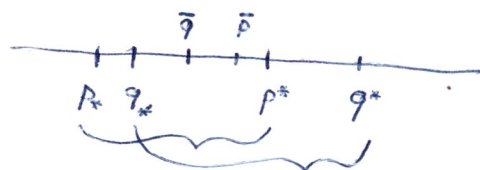
$P_{1*} < P_{2*} \Rightarrow b_{1*} < b_{2*}$

$P_* = P^* \Rightarrow b_* = b^*$

(but b_* may = b^* although $P_* < P^*$; in general, $P_* \leq b_* \leq b^* \leq P^*$)

Savage assumes $b_* = b^*$, without committing himself on $P_* = P^*$.

Even with this, "betting quotients" might vary independently of intuitive probs



In this case (as with VN-M utilities vs. "intuitive" utilities) it is betting probs. in which Savage is interested. Don't "explain" ^{or try to "correct"} choices by reference to beliefs, but by reference to other choices, in the light of the axioms.

Knowing b_* and b^* , can predict the acceptability of any bet, vs. no-bet.

What if $b_* - b^*$ overlaps with $b'_* - b'^*$?

$\begin{matrix} E & \bar{E} \\ (3 & -1) = 0 \end{matrix} \quad \begin{matrix} F & \bar{F} \\ (2 & -1) = 0 \end{matrix}$

Can predict any bets (in utils) on E (using b_*) or against E (using b^*).

$\begin{matrix} (2 & -1) = -\frac{1}{4} & (1 & -1) = -\frac{1}{3} \end{matrix}$

$\begin{matrix} (-4 & 2) = -2\frac{1}{2} & (-1 & -2) = -\frac{5}{3} \end{matrix}$

$\begin{matrix} (4 & 1) = 1\frac{3}{4} & (5 & 2) = 3 \end{matrix}$

For more than 2 events, construct least favorable dist